

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR
(AUTONOMOUS)

B.Tech. I Year I Semester Regular & Supplementary Examinations December/January-2025/2026
LINEAR ALGEBRA & CALCULUS

(Common to All)

Time: 3 Hours

Max. Marks: 70

PART-A

(Answer all the Questions 10 x 2 = 20 Marks)

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|-----|---|-----|----|----|
| 1 a | Define rank of the matrix. | CO1 | L1 | 2M |
| b | State Cauchy-Binet formulae. | CO1 | L1 | 2M |
| c | Find the Eigen Values of the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$. | CO2 | L3 | 2M |
| d | Find the symmetric matrix corresponding to the quadratic form $ax^2 + 2hxy + by^2$. | CO2 | L3 | 2M |
| e | State Lagrange's mean value theorem. | CO3 | L1 | 2M |
| f | Expand Taylor's series of the function $f(x)$ in powers of $(x-a)$. | CO4 | L2 | 2M |
| g | Evaluate $\lim_{x \rightarrow 1} \frac{2x^2y}{x^2+y^2+1}$. | CO5 | L5 | 2M |
| h | Define Extreme value of a function of two variables. | CO5 | L1 | 2M |
| i | Evaluate $\int_0^2 \int_0^x y \, dy \, dx$. | CO6 | L5 | 2M |
| j | Find the area enclosed by the parabolas $x^2 = y$ and $y^2 = x$. | CO6 | L1 | 2M |

PART-B

(Answer all Five Units 5 x 10 = 50 Marks)

UNIT-I

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|-----|---|-----|----|----|
| 2 a | Reduce the matrix $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ into Echelon form and find its rank. | CO1 | L3 | 5M |
| b | Solve completely the system of equations $4x + 2y + z + 3w = 0$; $6x + 3y + 4z + 7w = 0$; $2x + y + w = 0$. | CO1 | L3 | 5M |

OR

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|---|--|-----|----|-----|
| 3 | Express the following system in matrix form and solve by Gauss elimination method.
$2x_1 + x_2 + 2x_3 + x_4 = 6$; $6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$;
$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$; $2x_1 + 2x_2 - x_3 + x_4 = 10$. | CO1 | L2 | 10M |
|---|--|-----|----|-----|

UNIT-II

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|---|---|-----|----|-----|
| 4 | Find the Eigen values and the corresponding Eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. | CO2 | L3 | 10M |
|---|---|-----|----|-----|

OR

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|---|--|-----|----|-----|
| 5 | Reduce the quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$ in to the canonical form by an orthogonal transformation and discuss it's nature. | CO2 | L3 | 10M |
|---|--|-----|----|-----|

UNIT-III

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|-----|--|-----|----|-----|
| 6 a | Verify Rolle's theorem for the function $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$. | CO3 | L2 | 5M |
| b | Verify Lagrange's mean value theorem for $f(x) = \log_e x$ in $[1, e]$. | CO3 | L2 | 5M |
| OR | | | | |
| 7 | Using Maclaurin's series expand $\tan x$ up to the fifth power of x and hence find the series for $\log(\sec x)$. | CO4 | L3 | 10M |

UNIT-IV

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|-----|--|-----|----|-----|
| 8 a | If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$. | CO5 | L5 | 5M |
| b | If $u = \tan^{-1} \left[\frac{2xy}{x^2 - y^2} \right]$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. | CO5 | L5 | 5M |
| OR | | | | |
| 9 | Examine the maxima and minima, if any of the function $f(x, y) = x^3 y^2 (1 - x - y)$. | CO5 | L4 | 10M |

UNIT-V

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|------|---|-----|----|----|
| 10 a | Evaluate $\iint (x^2 + y^2) \, dx \, dy$ in the positive quadrant for which $x + y \leq 1$. | CO6 | L5 | 5M |
| b | Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z} \, dx \, dy \, dz$. | CO6 | L5 | 5M |
| OR | | | | |
| 11 a | Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. | CO6 | L1 | 5M |
| b | Calculate the volume of the solid bounded by the planes $x = 0$, $y = 0$, $x + y + z = a$ and $z = 0$. | CO6 | L1 | 5M |

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